

# Relativistic effects in ternary fission

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**Abstract.** Relativistic effects can explain the energy shift of  $-6$  keV reported by Ramayya *et al.* for the  $\gamma$ -ray de-exciting the first  $2^+$  state, at 3368.03 keV, of the  $^{10}\text{Be}$  cluster emitted together with the fragments  $^{146}\text{Ba}$  and  $^{96}\text{Sr}$  in the ternary fission of  $^{252}\text{Cf}$ ; the calculated shift is equal to  $-6.14$  (0.16) keV. An explanation is presented for the apparent absence of Doppler broadening. For the configurations  $^{138}\text{Xe}$ - $^{104}\text{Zr}$ - $^{10}\text{Be}$  and  $^{136}\text{Te}$ - $^{106}\text{Mo}$ - $^{10}\text{Be}$ , recently reported by Hamilton *et al.*, observable shifts of the 3367.43 keV  $\gamma$ -line are predicted.

**PACS.** 03.30.+p Special relativity – 23.20.-g Electromagnetic transitions – 25.85.-w Fission reactions

## 1 Introduction

The emission of clusters as heavy as  $^{10}\text{Be}$  in the ternary fission of  $^{252}\text{Cf}$  has been observed by Singer *et al.* [1]; these authors reported that the 3367.43 keV gamma-ray de-exciting the first  $2^+$  level at 3368.03 keV [2] for this ternary light charged particle (LCP) does not exhibit a Doppler effect [3]. Using the Gammasphere detector, Ramayya *et al.* [4] have found that the energy of this gamma-ray, measured in coincidence with the fission partners  $^{146}\text{Ba}$  and  $^{96}\text{Sr}$ , is lowered by 6 keV as compared with the value of the unperturbed nucleus. According to Misicu *et al.* [5], the energy shift could result from a perturbation of the  $^{10}\text{Be}$  nucleus by the two main fragments of the ternary configuration. An alternative explanation is that this shift results from a relativistic effect [6].

The aim of this paper is to show that all modes of the ternary LCP emission involve great LCP energies. Consequently, electromagnetic radiations emitted by ternary LCPs are necessarily affected by relativistic effects, such as the time dilatation effect. The aim of this paper is further to show that this last effect can be put into evidence even in the so-called “low-energy” LCP emission mode, and that it can explain the 6 keV energy shift observed by Ramayya *et al.*, or lead to predict a variety of observable energy shifts for recently observed “cold” ternary configurations [7] such as  $^{138}\text{Xe}$ - $^{104}\text{Zr}$ - $^{10}\text{Be}$  and  $^{136}\text{Te}$ - $^{106}\text{Mo}$ - $^{10}\text{Be}$ .

Finally, the aim of this paper is to suggest an explanation of the apparent absence of Doppler broadening of the 3367.43 keV  $\gamma$ -ray, though its energy is lowered by a relativistic effect.

## 2 The time dilatation effect

If a cluster is emitted with a great kinetic energy, the frequency  $\nu'_{\text{source}}$  of the radiation emitted by the LCP is reduced to a value

$$\nu_{\text{observer}} = \nu'_{\text{source}} \sqrt{1 - \frac{v^2}{c^2}}, \quad (1)$$

where  $v$  is the velocity of the LCP. It is a consequence of Einstein’s “time dilatation”.

According to this relation, an energy shift of  $-6$  keV has to be expected for a 3367.43 keV  $\gamma$ -line if it is emitted by an excited  $^{10}\text{Be}^*$  LCP having a kinetic energy of 16.6 MeV<sup>1</sup>.

And an even greater negative energy shift has to be expected for the same  $\gamma$ -line, if it is emitted by a  $^{10}\text{Be}^*$  LCP having an even greater kinetic energy.

The relation giving the relativistic Doppler effect [8] becomes reduced to eq. (1) if the factor  $(1 - \frac{v}{c} \cos \alpha)^{-1}$ , where  $\alpha$  is the angle between the velocity  $\vec{v}_{\text{source}}$  of the emitting LCP and the velocity  $\vec{v}_{\text{observer}}$  of the observer, becomes equal to 1, *i.e.* if  $\alpha$  becomes equal to  $\pi/2$  or  $3\pi/2$ .

## 3 The two emission modes of ternary LCPs

In 1996, an explanation of the low-energy “orthogonal”  $\alpha$ -particle emission of ternary fission has been

<sup>1</sup> If  $\sqrt{1 - \frac{v^2}{c^2}} = 3361.43/3367.43 = 0.9982182$ ,  $v^2 = 3.19985 \cdot 10^{14} \text{ m}^2 \text{ s}^{-2}$  and  $\text{KE} (^{10}\text{Be}^*) = 16.578 \text{ MeV}$ .

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proposed [9,10]: This emission could result from a stimulation of the latent  $\alpha$ -radioactivity of fission fragments by the double giant dipole resonance (DGDR) associated with the fission reaction.

In 1998, an explanation of the higher-energy, “isotropic”  $\alpha$ -particle or LCP emission of ternary fission has been proposed [11]: This second mode could result from the clusterization of a fission fragment, followed by a kind of collision of this fragment with its fission partner. It explains the high-energy  $\alpha$ -particle emission first observed by Piasecki *et al.* [12] and carefully studied by Heeg [13]; due to the dominant low-energy, orthogonal emission mode, the high-energy one was first observed only in the direction of the fission axis, and called “polar” emission mode for this reason. But the high-energy mode explains also the “isotropic” emission of LCPs first observed by Chen *et al.* [14] in fusion-fission reactions.

#### 4 The energy shift of $\sim -6$ keV of the $^{146}\text{Ba}-^{96}\text{Sr}-^{10}\text{Be}$ configuration of $^{252}\text{Cf}$

It is reasonable to assume that a giant dipole resonance occurs in *each* preformed fragment. Its energy is given by the relation

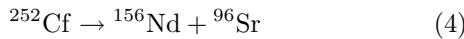
$$E_{\text{GDR}} = 31.2A^{-1/3} + 20.6A^{-1/6}, \quad (2)$$

according to Berman and Fultz [15], and the double giant dipole resonance has an energy given by the relation

$$E_{\text{DGDR}} = 2E_{\text{GDR}}, \quad (3)$$

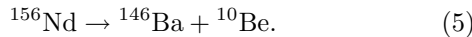
according to Emling [16].

Let us consider a  $^{156}\text{Nd}$  fragment, formed according to



in the binary fission of  $^{252}\text{Cf}$ .

This fragment can clusterize according to



By analogy with the  $Q_\alpha$  of  $\alpha$ -particles tabulated by ref. [17], the  $Q_{^{10}\text{Be}}$  can be calculated as a difference of binding energies, here,

$$Q_{^{10}\text{Be}} = E_{\text{B}}(^{10}\text{Be}) - [E_{\text{B}}(^{156}\text{Nd}) - E_{\text{B}}(^{146}\text{Ba})]; \quad (6)$$

using experimental mass data [17],  $Q_{^{10}\text{Be}}$  is found equal to  $-7.86$  (0.48) MeV.

Noteworthy is the minus sign<sup>2</sup>; it is the reason of the latent character of the “ $^{10}\text{Be}$ -radioactivity”: indeed, without a stimulation by the DGDR, no  $^{10}\text{Be}$  cluster could be emitted by most of the fission fragments.

Noteworthy, too, is the considerable lack of precision of the mass data involved in eq. (6); this situation is not an

<sup>2</sup> Positive  $Q_{\text{LCP}}$ 's are exceptional; an example is given by  $Q_{^{34}\text{Si}}(^{166}\text{Gd})$ , equal to 42.15 MeV, due to the great cohesion of the  $^{132}\text{Sn}$  and  $^{34}\text{Si}$  nuclei, cf. ref. [18]. Positive  $Q_{^{10}\text{Be}}$ 's in ref. [6] were a mistake.

exception; for this reason, from now on we will neglect all causes of error other than those resulting from the lack of precision of the mass data relative to any fission fragment; even the error resulting from the use of the non-relativistic expression of the kinetic energy has been neglected (see, nevertheless, the caption of tables 1 and 2).

According to eqs. (2) and (3), the DGDR energy of the  $^{156}\text{Nd}$  fragment is equal to 29.35 MeV; and that of the  $^{106}\text{Mo}$  fragment is equal to 32.12 MeV.

As soon as the  $^{156}\text{Nd}$  fragment obtains energy from the DGDR, the available energy in this fragment becomes equal to  $E_{\text{DGDR}} + Q_{^{10}\text{Be}} = 21.49$  (0.48) MeV. And if the clusterized fragment dissociates, the kinetic energy of  $^{10}\text{Be}$  can be equal to

$$\text{KE}(^{10}\text{Be}) = \frac{146}{156}(21.49 \pm 0.48) = (20.11 \pm 0.45) \text{ MeV}. \quad (7)$$

This value is reported in table 1, together with the  $Q_{^{10}\text{Be}}$ .

For the  $^{106}\text{Mo}$  fragment, clusterized according to  $^{96}\text{Sr} + ^{10}\text{Be}$ , similar calculations lead to  $Q_{^{10}\text{Be}} = -15.910$  (0.047) MeV and to  $\text{KE}(^{10}\text{Be}) = 14.68$  (0.04) MeV.

The results, 20.11 MeV and 14.68 MeV, obtained for the kinetic energy of the  $^{10}\text{Be}$  ternary particles, considered in their ground state, of the configuration  $^{146}\text{Ba}-^{96}\text{Sr}-^{10}\text{Be}$ , can be considered as very satisfactory. Indeed, the “mean” kinetic energy of all Be particles emitted by  $^{252}\text{Cf}$  has been found equal to 17.5 MeV in the experiments of Mutterer *et al.* [19], and  $^{10}\text{Be}$  represents 80% of all the ternary Be particles of  $^{252}\text{Cf}$ .

With the value of 20.11 MeV for the kinetic energy of  $^{10}\text{Be}$ , the square of the velocity is equal to

$$v_{^{10}\text{Be}}^2 = \frac{2 \times 20.11 \times 1.602 \times 10^{-13} \text{ kg m}^2 \text{ s}^{-2}}{10 \times 1.660 \times 10^{-27} \text{ kg}} = 3.88147 \cdot 10^{14} \text{ m}^2 \text{ s}^{-2}. \quad (8)$$

With  $c = 2.9979 \cdot 10^8 \text{ m s}^{-1}$ , it corresponds to  $v = 0.066c$ .

This 20.11 MeV energy and the corresponding velocity  $0.066c$  correspond to  $^{10}\text{Be}$  LCPs formed in their ground state. But the 3367.43 keV  $\gamma$ -line is emitted by  $^{10}\text{Be}^*$  LCPs formed in their first  $2^+$  excited state at 3368.03 keV. For their formation, the DGDR has to furnish more than 7.86 (0.48) MeV, namely  $7.86 + 3.368$  MeV, as if the true  $Q_{^{10}\text{Be}}$  were  $Q_{^{10}\text{Be}^*} = -11.23$  (0.48) MeV. Thus, the internal excitation energy of the  $^{156}\text{Nd}$  fragment clusterized according to  $^{146}\text{Ba} + ^{10}\text{Be}^*$  is 18.12 (0.48) MeV, rather than 21.49 MeV. The corresponding KE of ( $^{10}\text{Be}^*$ ) is then

$$\text{KE}(^{10}\text{Be}^{2+}) = \frac{146}{156}(18.12) = (16.96 \pm 0.45) \text{ MeV}. \quad (7\text{bis})$$

For the  $^{106}\text{Mo}$  fragment, similar calculations lead to

$$\text{KE}(^{10}\text{Be}^{2+}) = 11.63 (0.04) \text{ MeV}.$$

The corresponding quadratic velocity of the  $^{10}\text{Be}^*$  ( $^{156}\text{Nd}$ ) is now

$$v^2 = 3.273484 \cdot 10^{14} \text{ m}^2 \text{ s}^{-2} \quad (8\text{bis})$$

**Table 1.** Predicted kinetic energies of the  $^{10}\text{Be}$  LCPs in the ground state (columns 4 and 7) and in the first-excited state (columns 5 and 8), and predicted energy shifts  $h\Delta\nu$  of the 3367.43 keV  $\gamma$ -line (columns 6 and 9) for the low-energy mode and for the high-energy mode for three ternary mass splits of  $^{252}\text{Cf}$  (column 1). The use of the non-relativistic expression of the kinetic energy leads to a (not given) extra error on KE ( $^{10}\text{Be}^{2+}$ ), which is smaller than 1% as long as KE is smaller than 62 MeV, and to an extra error on the corresponding  $h\Delta\nu$ , which is smaller than 0.37 keV in this case.

			LCP EMISSION MODE					
			Low-energy $^{10}\text{Be}$ emission			High-energy $^{10}\text{Be}$ emission		
1	2	3	4	5	6	7	8	9
Ternary mass split	Binary mass split & clusterization	$Q_{^{10}\text{Be}}$	KE $^{10}\text{Be}$ (g.s.) (MeV)	KE $^{10}\text{Be}$ ( $2_1^+$ ) (MeV)	$h\Delta\nu_{\text{theor.}}$ (keV)	KE $^{10}\text{Be}$ (g.s.) (MeV)	KE $^{10}\text{Be}$ ( $2_1^+$ ) (MeV)	$h\Delta\nu_{\text{theor.}}$ (keV)
$^{146}\text{Ba}$ $^{96}\text{Sr}$ $^{10}\text{Be}$	$^{156}\text{Nd} + ^{96}\text{Sr}$ $^{156}\text{Nd} \rightarrow ^{146}\text{Ba} + ^{10}\text{Be}$	-7.86 $\pm 0.48$ MeV	20.11 $\pm 0.45$	16.96 $\pm 0.45$	-6.14 $\pm 0.16$	67.28 $\pm 0.60$	64.17 $\pm 0.60$	-23.29 $\pm 0.22$
	$^{146}\text{Ba} + ^{106}\text{Mo}$ $^{106}\text{Mo} \rightarrow ^{96}\text{Sr} + ^{10}\text{Be}$	-15910 $\pm 47$ keV	14.68 $\pm 0.04$	11.63 $\pm 0.04$	-4.209 $\pm 0.015$	99.66 $\pm 0.10$	96.53 $\pm 0.10$	-35.09 $\pm 0.04$
$^{138}\text{Te}$ $^{104}\text{Mo}$ $^{10}\text{Be}$	$^{148}\text{Ba} + ^{104}\text{Mo}$ $^{148}\text{Ba} \rightarrow ^{138}\text{Te} + ^{10}\text{Be}$	-4.72 $\pm 0.35$ MeV	23.30 $\pm 0.33$	20.16 $\pm 0.33$	-8.43 $\pm 0.12$	78.10 $\pm 0.40$	<b>74.96</b> $\pm 0.40$	<b>-27.22</b> $\pm 0.15$
	$^{138}\text{Te} + ^{114}\text{Pd}$ $^{114}\text{Pd} \rightarrow ^{104}\text{Mo} + ^{10}\text{Be}$	-15770 $\pm 44$ keV	14.42 $\pm 0.04$	11.35 $\pm 0.04$	-4.107 $\pm 0.014$	98.24 $\pm 0.20$	95.17 $\pm 0.20$	-34.59 $\pm 0.07$
$^{134}\text{Te}$ $^{108}\text{Mo}$ $^{10}\text{Be}$	$^{144}\text{Ba} + ^{108}\text{Mo}$ $^{144}\text{Ba} \rightarrow ^{134}\text{Te} + ^{10}\text{Be}$	-1992 $\pm 44$ keV	25.97 $\pm 0.04$	22.84 $\pm 0.04$	-8.27 $\pm 0.21$	85.48 $\pm 0.21$	82.35 $\pm 0.21$	-29.91 $\pm 0.08$
	$^{134}\text{Te} + ^{118}\text{Pd}$ $^{118}\text{Pd} \rightarrow ^{108}\text{Mo} + ^{10}\text{Be}$	-16.88 $\pm 0.41$ MeV	13.22 $\pm 0.37$	10.13 $\pm 0.37$	-3.67 $\pm 0.14$	98.38 $\pm 0.56$	95.29 $\pm 0.56$	-34.64 $\pm 0.20$

**Table 2.** Prediction of the energy shift  $h\Delta\nu_{\text{theor.}}$  of the 3367.43 keV  $\gamma$ -line transition of the  $^{10}\text{Be}$  cluster appearing in the ternary mass splits  $^{136}\text{Te}$ - $^{106}\text{Mo}$ - $^{10}\text{Be}$  and  $^{138}\text{Xe}$ - $^{104}\text{Zr}$ - $^{10}\text{Be}$  of  $^{252}\text{Cf}$ . The use of the non-relativistic expression of the kinetic energy leads to a (not given) extra error on KE ( $^{10}\text{Be}^{2+}$ ), which is smaller than 1% as long as KE is smaller than 62 MeV, and to an extra error on the corresponding  $h\Delta\nu$ , which is smaller than 0.37 keV in this case.

			LCP EMISSION MODE					
			Low-energy $^{10}\text{Be}$ emission			High-energy $^{10}\text{Be}$ emission		
1	2	3	4	5	6	7	8	9
Ternary mass split	Binary mass split & clusterization	$Q_{^{10}\text{Be}}$	KE $^{10}\text{Be}$ (g.s.) (MeV)	KE $^{10}\text{Be}$ ( $2_1^+$ ) (MeV)	$h\Delta\nu_{\text{theor.}}$ (keV)	KE $^{10}\text{Be}$ (g.s.) (MeV)	KE $^{10}\text{Be}$ ( $2_1^+$ ) (MeV)	$h\Delta\nu_{\text{theor.}}$ (keV)
$^{136}\text{Te}$ $^{106}\text{Mo}$ $^{10}\text{Be}$	$^{146}\text{Ba} + ^{106}\text{Mo}$ $^{146}\text{Ba} \rightarrow ^{136}\text{Te} + ^{10}\text{Be}$	-3.29 $\pm 0.13$	24.70 $\pm 0.13$	21.56 $\pm 0.13$	-7.804 $\pm 0.047$	82.11 $\pm 0.16$	78.98 $\pm 0.16$	-28.68 $\pm 0.06$
	$^{116}\text{Pd} + ^{136}\text{Te}$ $^{116}\text{Pd} \rightarrow ^{106}\text{Mo} + ^{10}\text{Be}$	-16.314 $\pm 0.082$	13.832 $\pm 0.075$	10.754 $\pm 0.075$	-3.891 $\pm 0.027$	98.72 $\pm 0.13$	95.65 $\pm 0.13$	-34.76 $\pm 0.05$
$^{138}\text{Xe}$ $^{104}\text{Zr}$ $^{10}\text{Be}$	$^{148}\text{Ce} + ^{104}\text{Zr}$ $^{148}\text{Ce} \rightarrow ^{138}\text{Xe} + ^{10}\text{Be}$	-2.92 $\pm 0.16$	24.98 $\pm 0.15$	21.84 $\pm 0.15$	-7.905 $\pm 0.054$	79.16 $\pm 0.35$	76.02 $\pm 0.35$	-27.60 $\pm 0.13$
	$^{138}\text{Xe} + ^{114}\text{Ru}$ $^{114}\text{Ru} \rightarrow ^{104}\text{Zr} + ^{10}\text{Be}$	-17.63 $\pm 0.76$	13.24 $\pm 0.69$	10.17 $\pm 0.69$	-8.68 $\pm 0.25$	97.81 $\pm 0.90$	94.71 $\pm 0.90$	-34.42 $\pm 0.33$

and the velocity  $v = 0.060c$ .

With  $\sqrt{1 - \frac{v^2}{c^2}} = 0.9981772$ , the energy shift is

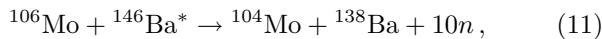
$$h\Delta\nu = -6.14(0.16) \text{ keV}. \quad (9)$$

For the  $^{106}\text{Mo}$  fragment, similar calculations lead to a shift

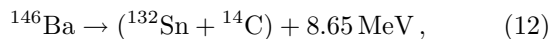
$$h\Delta\nu = -4.21(0.02) \text{ keV}. \quad (10)$$

It is noteworthy that the shift (9) is in perfect agreement with the experimental value reported by Ramayya *et al.* [4], since  $3368 - 3362(4) = 6 \text{ keV}$ .

Furthermore, a tentative analysis of the coincidence  $\gamma$ -spectrum of fig. 2 of ref. [4] shows that the contribution, to the measured shift, of the  $-4.21 \text{ keV}$  shift resulting from the clusterization of the preformed  $^{106}\text{Mo}$  fragment cannot be important. Such a situation could be explained, either by a value of the fission yield of the binary mass split  $^{146}\text{Ba}-^{106}\text{Mo}$  smaller than that of the binary mass split  $^{156}\text{Nd}-^{96}\text{Sr}$ , or by the competition of processes such as



*i.e.* by the competition of one of the various aspects of the hot-fission mode [20]. Indeed, eq. (11) is a consequence of an energy-yielding clusterization process:



followed by the destruction of the “deformed” (because clusterized)  $^{146}\text{Ba}^*$  fragment in a pre-scission collision with its fission partner  $^{106}\text{Mo}$ , for example according to reaction (11). The reality of a process such as eq. (11) has been confirmed by Hamilton *et al.* [7]. And the explanation of the hot-fission process by the clusterization of a preformed fragment has been suggested by Mouze and Ricci [21].

## 5 Discussion

### 5.1 The apparent absence of Doppler broadening in the Gammasphere experiment

Let us consider the coincidence  $\gamma$ -ray spectrum shown in fig. 2 of ref. [4]. The width at half-maximum of the  $3362(4) \text{ keV}$  line is equal to  $9 \text{ keV}$ , *i.e.* to 27 channels, the slope being of  $0.3333 \text{ keV}$  per channel. The narrowness of this line is surprising, because  $^{10}\text{Be}$  particles in their excited state are expected to have a velocity of  $0.060c$ , according to eq. (8bis), and the emitted  $\gamma$ -ray is expected to have a Doppler broadening of about  $\pm 202 \text{ keV}$ , *i.e.*  $\pm 606$  channels. However, if the  $3362 \text{ keV}$  line were displayed over 1212 channels, it would be hardly observable, as a consequence of the poor statistics, since *the reported number of counts for the shifted line itself, at 3362 keV, is only 2.2 counts in each of the involved 27 channels*, and since the background has not been subtracted from the spectrum of fig. 2. Inversely, a careful examination of this spectrum shows that *the maximum number of counts attributable to*

*a Doppler distribution having a width of 1212 channels is about 0.2 count per channel*. Thus, the broadened line, if it nevertheless is present, could correspond to about 242 counts, *i.e.* four times the number of counts observed in the  $3362 \text{ keV}$  peak (59 counts). This last observation suggests that we are faced with two phenomena in apparent competition.

### 5.2 Is it possible to explain the narrowness of the observed line at 3362 keV?

As indicated in sect. 2, the only way of observing the emitted  $\gamma$ -ray without broadening would be to limit the conditions of observation to an observation at ninety degrees from the direction of propagation of the  $^{10}\text{Be}$  LCPs. But how could this condition be fulfilled at Gammasphere?

Let us recall that the low-energy ternary LCP emission occurs at 90 degrees from the fission axis. This means that there is no coupling between the motion of the emitted cluster and the motion of the fragments along the fission axis; only a recoil effect is expected for the emitting fragment; indeed, the kinetic energy of the emitting fragment along the fission axis did not play a role in our calculations; only the value of  $Q_{^{10}\text{Be}}$  and the value of  $E_{\text{DGDR}}$  seem to play a role in the formation of the ternary “orthogonal” LCP (the case of the “isotropic”, high-energy LCP emission mode is different: there, the motion of the emitting fragment along the fission axis plays the major role, as will be shown in sect. 6).

Let us recall also *the existence of a giant dipole resonance in each fragment*; it results from the out-of-phase oscillation of protons and neutrons. The existence of these *two oscillating dipoles* already explains, by the distribution of their electric lines of force, the emission of the low-energy LCPs in a direction almost perpendicular to the fission axis. Thus *their velocity vector is confined in a plane which is almost perpendicular to the fission axis*.

Finally, let us consider *gamma-rays emitted in a direction perpendicular to this plane*. If these gamma-rays are received by a detector of the Gammasphere device, they should not be Doppler broadened; only their energy is changed.

This means that each detector of the Gammasphere has the same small but well determined chance of giving an energy-shifted but not broadened line, with an energy of  $3361.3 \text{ keV}$  in the case studied by Ramayya *et al.*

## 6 Energy shifts of the high-energy emission mode

Recently, Hamilton *et al.* [7] reported that the  $^{134}\text{Te}-^{108}\text{Mo}-^{10}\text{Be}$  ternary configuration of  $^{252}\text{Cf}$ , for which an energy shift of about  $-26 \text{ keV}$  had been reported [22], but not published, could be a  $^{138}\text{Te}-^{104}\text{Mo}-^{10}\text{Be}$  configuration as well; the reason is “that  $^{108}\text{Mo}$  and  $^{104}\text{Mo}$  have  $2^+ \rightarrow 0^+$  transitions that are too close in energy to resolve, and that their  $4^+ \rightarrow 2^+$  transitions are barely resolvable with peak-fitting routines” [7].

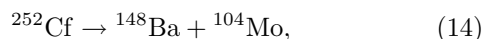
Let us remark that an energy shift as great as  $-26$  keV cannot be caused by a  $^{10}\text{Be}$  cluster emitted according to the low-energy mode. Such an energy shift clearly indicates that the  $^{10}\text{Be}$  cluster must have been emitted with a considerable velocity,  $v = 0.124c$ , and with a kinetic energy as great as  $71.61$  MeV: Only the high-energy mode can be responsible for such a velocity and such a kinetic energy.

Let us now compare the energy shifts predicted for the configurations  $^{134}\text{Te}-^{108}\text{Mo}-^{10}\text{Be}$  and  $^{138}\text{Te}-^{104}\text{Mo}-^{10}\text{Be}$  in the high-energy case. Table 1 shows that the  $^{138}\text{Te}-^{104}\text{Mo}-^{10}\text{Be}$  configuration can be responsible for an energy shift close to  $-26$  keV; indeed, if  $^{138}\text{Te}$  is formed in the clusterization of  $^{148}\text{Ba}$  according to



with  $Q_{^{10}\text{Be}} = -4.72$  (0.35) MeV, and an apparent  $Q_{^{10}\text{Be}^*}$  of  $-8.088$  MeV for the formation of  $^{10}\text{Be}^{2+}$ , a theoretical shift of  $-27.22$  (0.15) keV can be expected, because the kinetic energy of the  $^{10}\text{Be}^*$  cluster can be equal to  $74.96$  (0.40) MeV.

In fact,  $^{148}\text{Ba}$  is formed in the binary fission



with  $Q_{\text{tot.}} = 214.40$  (0.20) MeV, and an excitation energy of  $88.48$  MeV, but after clusterization leading to the formation of  $^{10}\text{Be}^{2+}$  its energy becomes  $80.39$  (0.43) MeV. Thus, in a kind of collision with its fission partner  $^{104}\text{Mo}$ ,  $^{148}\text{Ba}^*$  can emit a  $^{10}\text{Be}$  cluster with this energy of  $74.96$  MeV.

Table 1 shows that no agreement with the reported shift of about  $-26$  keV can be obtained for the  $^{134}\text{Te}-^{108}\text{Mo}-^{10}\text{Be}$  configuration, if correct negative  $Q_{^{10}\text{Be}}$ 's are used.

## 7 New predictions

No energy shifts have been measured until now for the recently confirmed cold ternary configurations  $^{136}\text{Te}-^{106}\text{Mo}-^{10}\text{Be}$  and  $^{138}\text{Xe}-^{104}\text{Zr}-^{10}\text{Be}$ . However, several energy shifts values can be predicted, as shown in table 2. It must be pointed out that the possibility of observing these predicted shifts depends on the yields of the involved binary fragments, on the resolution of the spectrometer and on the existence of destructive clusterization processes, such as those involved in the hot-fission mode.

## 8 Conclusion

The discussion of the  $-6$  keV shift of the  $3367.43$  keV radiation of the  $^{146}\text{Ba}-^{96}\text{Sr}-^{10}\text{Be}$  configuration of  $^{252}\text{Cf}$  has shown that the time dilatation effect of the theory of relativity can explain this energy shift and furnishes a theoretical value very close to the experimental one.

The observation of the shifted line as a narrow line is a consequence of the laws of the relativistic Doppler effect, the existence of a "second-order" effect causing geometrical probabilities to come into play. The contribution of the first-order Doppler effect is hardly observable as a consequence of the great value of  $v/c$ , and thus of the broadening.

At the same time, the hypothesis of the stimulation of the LCP emission mode by the DGDR associated with the  $^{252}\text{Cf}$  fission has led to a kinetic-energy value of the emitting  $^{10}\text{Be}$  cluster in excellent agreement with the mean kinetic energy of the  $^{10}\text{Be}$  particles accompanying the fission of  $^{252}\text{Cf}$ .

If correct negative  $Q_{^{10}\text{Be}}$  values are used, no agreement is found between theory and experiment for the  $^{134}\text{Te}-^{108}\text{Mo}-^{10}\text{Be}$  configuration, whereas an energy shift of  $-27.22$  keV is predicted for the  $^{138}\text{Te}-^{104}\text{Mo}-^{10}\text{Be}$  configuration, differing by only 4.7% of a  $-26$  keV shift.

For the recently observed cold ternary configurations  $^{138}\text{Xe}-^{104}\text{Zr}-^{10}\text{Be}$  and  $^{136}\text{Te}-^{106}\text{Mo}-^{10}\text{Be}$ , several predictions of observable effects are presented, corresponding to the various clusterizations and cluster emission modes.

### Additional remark

The present paper rectifies and completes a previous paper on the same subject [6].

Observation of a shifted  $\gamma$ -line without broadening by the dominant first-order Doppler effect is possible only in coincidence with a particular ternary configuration, as in Ramayya's experiment [4]. This explains the negative result of A.V. Daniel *et al.* [23].

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